Deep Learning for CT Reconstructions

CT Code Sprint 2020

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15.06.2020

European Commission

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Code Sprint 2020

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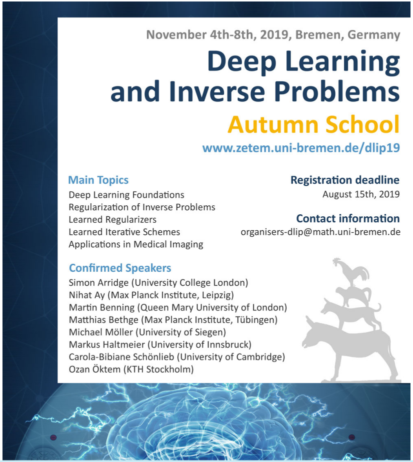
Table of Contents

Code Sprint



CT Challenge
M. van Eijnatten

June 15 - 24, 2020, Bremen

- Compare network designs
- Runtime
- Hardware requirements

DL for tomography

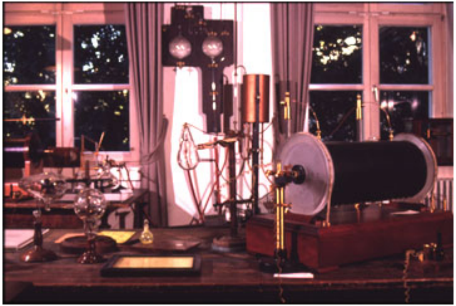
- More complex challenges
- Extension to dynamic CT

From physics to math: X-ray technology

- Model
- Abstraction
- Theorie
- Applications

Wilhelm Conrad Röntgen (*1845 – †1923)

W.C. Röntgen, 22.12.1895, Deutsches Museum

Tomography

- E_0 X-ray Source
- E_L measured photons
- attentuation $\triangleq$ density
- rotation

Tomography

- E_0 X-ray Source
- E_L measured photons
- attentuation $\hat{=}$ density
- rotation

Mathematics of CT

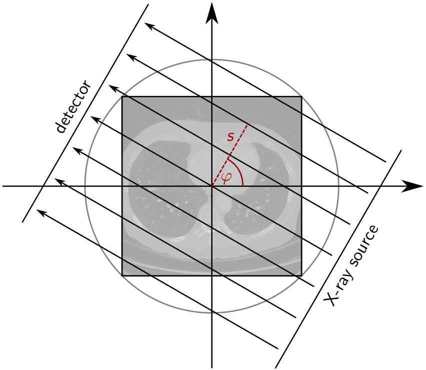


Figure: Parallel beam geometry

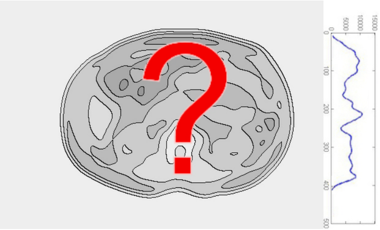
- Radon transform $Ax(s,\varphi)$ simulates the attenuation of a single beam

$$-\ln\left(\frac{E_L}{E_0}\right) = Rf(s,\varphi) = \int_{\mathbb{R}} f\left(s\omega(\varphi) + t\omega^{\perp}(\varphi)\right) \mathrm{d}t$$

- Reconstruction formula (filtered back projection)

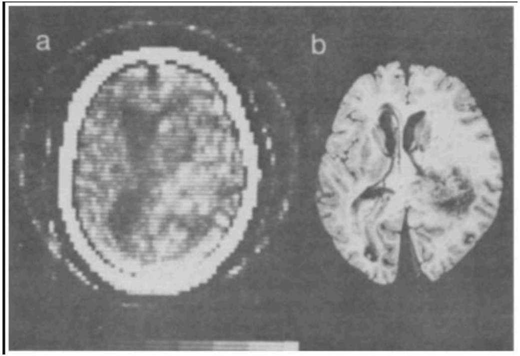
$$f(x) = \sqrt{2\pi} \int_{S^1} \int_{\boldsymbol{R}} e^{i\sigma\, x\cdot\omega}\, |\sigma|\, \widehat{Rf}\,(\sigma,\omega)\; d\sigma d\omega$$

Reconstruction



- 1917 J. Radon
- 1979 Cormack & Hounsfield
- 1982 F. Natterer
- > 2000 Applications, hybrid data and algorithms (A.K. Louis)
- > 2015 Deep Learning

Reconstruction (A. Cormack, G.N. Hounsfield)



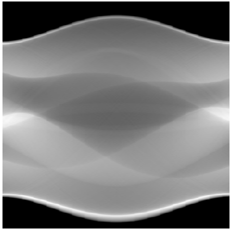
Linear system, sum over rows of pixels

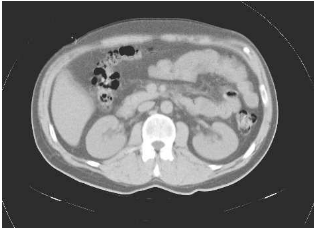
Reconstruction (F. Natterer)

$$= \mathcal{R}^{\#} I_1 \mathcal{R} f(x) = \sqrt{2\pi} \int_{S^1} \int_{\boldsymbol{R}} e^{i\sigma\, x\cdot\omega}\, |\sigma|\, \widehat{Rf}\, (\sigma,\omega)\, d\sigma d\omega$$

History of CT

Data driven image reconstruction, X-ray CT





SIEMENS, CT-tomograph, sinogram

A tomograph, measured data (sinogram) y^δ, x reconstruction

$$Ax \sim y^\delta$$

Inverse problem: given A, y^δ, determine x

Modelling - Simulation - Optimization
Scientific/ industrial application - mathematical model
Typical models: input (parameter) - output (state of a system)

- System of linear or non-linear equations
- Partial differential equations $u_t = div\sigma\nabla u$
- continuous or discrete, (non-)linear models

$$A : X \to Y, \quad X, Y \text{ function spaces}$$

Direct problem: Given input x, determine $y \sim Ax$
Inverse problem: given y determine x s.t. $Ax \sim y$

Model based approach

Model given by a matrix, differential equation, measurement, etc.

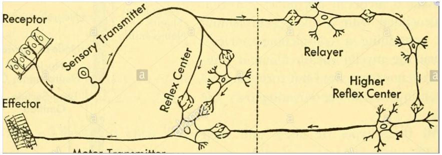
$$A : X \to Y,$$

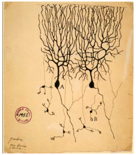
with A linear or non-linear "white model"

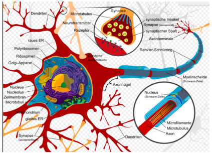
Drawbacks:

- every model is incomplete
- set of inputs $\mathcal{D}(A) \subset X$, unknown structure
- theory optimal for X general vector- or function space,
  e.g. in imaging (nature, human, radiological)
  Y. Meyer $X = \mathbb{R}^N$, $X = L^2(\Omega)$ or $X = \{v | v = div(z), z \in H^{\frac{3}{2}}(\Omega)\}$
  T. Pock, visualization of manifold of images
  S. Mallat, characterization by wavelet scatter transform
- How to include specific information on $\mathcal{D}(A)$?

The human neural system

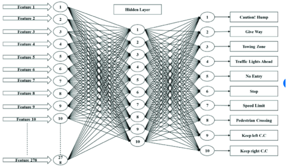


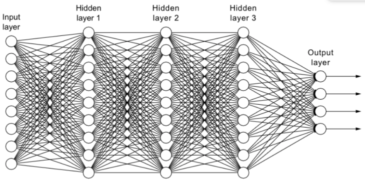




Number of neurons: 80.000.000.000

Length of pathways: 5.000.000 km





Formal definition: feedforward neural networks

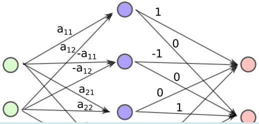
$$x_L = x_3 = \varphi(z_3)$$
$$z_3 = W_3 x_2 + b_3$$
$$x_2 = \varphi(z_2)$$
$$z_2 = W_2 x_1 + b_2$$
$$x_1 = \varphi_1(z_1)$$
$$z_1 = W_1 x_0 + b_1$$
$$x^0 = x$$

Feedforward neural network with L layers:

$$\begin{aligned} \text{input:} \quad & x_0 \\ & z_k = W_k x_{k-1} + b_k \\ & x_k = \varphi(z_k) \\ \text{ouput:} \quad & x_L \end{aligned}$$

φ non-linear, componentwise, network parameters $\mathcal{W} = \{W_k, b_k\}$
affine transformation, matrix W, bias b

$$\varphi_{\mathcal{W}}(x_0) = x_L = \varphi(W_3\ \varphi(W_2\ \varphi(W_1 x + b_1) + b_2) + b_3)$$



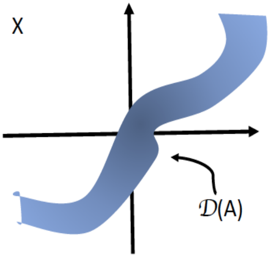
Deep learning concepts for inverse problems

"Every image is a matrix, but not every matrix is an image"

Postprocessing of analytic reconstructions

- Postprocessing, learned projection

$$\varphi : X \to \mathcal{D}(A)$$
$$y^\delta \to x \to \varphi(x)$$



Learned prior distribution

- probability for $x \in \mathcal{D}(A)$

$$min_x \; \|Ax - y^\delta\|^2 - log\left(\varphi(x)\right)$$

Deep learning concepts for inverse problems

"No model is perfect"

Learning operator updates

- Residual data $\left(x^{(d)}, y^{(d)} - Ax^{(d)}\right)$

$$\min_x \; \|Ax + \varphi(x) - y^\delta\|^2 + \alpha R(x)$$

Direct inversion A_α^{-1}

- fully learned inversion (main topic)

$$y^\delta \to \; x = \varphi(y^\delta)$$

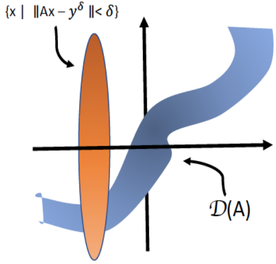
Review article: Arridge, PM, Öktem, Schönlieb, Acta Numerica 2019

Deep learning concepts for inverse problems

$Ax \sim y^\delta$, "No model is perfect", "Not every matrix is an image"

Training data (x_i, y_i^δ), $i = 1, N$, N large, neural network φ_W

- Postprocessing of classical reconstructions
- Forward operator $Ax + \varphi_W(x)$
  non-linear effects, efficiency
- Reconstruction $x_\alpha^\delta = \varphi_W(y^\delta)$
  Exploit prior distribution
- Learning Tikhonov penalty terms
  $\frac{1}{2}\|Ax - y^\delta\|^2 + \varphi_W(x)$



Mallat, Haltmeier, Adler, Öktem, Lunz, Schönlieb, Arridge, Hauptmann, Grasmaier, Harrach, Dittmer, Otero, ...

Overview DL Methods (incomplete list)

Iterative Methods:

- Adler et al., “Solving ill-posed inverse problems using iterative deep neural networks”
- Adler et al., “Learned primal-dual reconstruction”
- Gupta et al., “CNN-Based Projected Gradient Descent for Consistent CT Image Reconstruction”
- Hauptmann et al., “Multi-Scale Learned Iterative Reconstruction” [A. Hauptmann, S. Arridge]
- Corona et al., “Enhancing joint reconstruction and segmentation with non-convex Bregman iteration”
- Marlevi et al., “Multigrid Reconstruction in Tomographic Imaging” [M. Colarieti-Tosti]

Learned Regularization:

- Lunz et al., “Adversarial Regularizers in Inverse Problems” [S. Lunz]
- Li et al., “NETT: Solving Inverse Problems with Deep Neural Networks” [J. Schwab]
- Schwab et al., “Deep null space learning for inverse problems: convergence analysis and rates” [J. Schwab]

Conditional Distribution Sampling:

- Adler et al., “Deep Bayesian Inversion”
- Denker et al., *Conditional Normalizing Flows for Low-Dose Computed Tomography Image Reconstruction* [M. Schmidt, J. Leuschner, P. Maass]

Data-free Approaches:

- Baguer et al., *Computed Tomography Reconstruction Using Deep Image Prior and Learned Reconstruction Methods* [J. Leuschner, M. Schmidt]

Hybrid Approaches:

- Janssens et al., "Neural network Hilbert transform based filtered backprojection for fast inline x-ray inspection" [L. Alves Pereira]
- Bubba et al., "Learning the invisible: a hybrid deep learning-shearlet framework for limited angle computed tomography" [T. Bubba (Presentation 16.06.)]
- Wei et al., "Sparse-view CT image restoration via multiscale wavelet residual network" [X. Tao]

Post-Processing:

- Chen et al., "Low-Dose CT With a Residual Encoder-Decoder Convolutional Neural Network"
- Pelt et al., "Improving tomographic reconstruction from limited data using Mixed-Scale Dense convolutional neural networks" [D. Pelt]
- Hendriksen et al., "On-the-Fly Machine Learning for Improving Image Resolution in Tomography" [A. Hendriksen, D. Pelt, S. Coban]
- Etmann et al., *iUNets: Fully invertible U-Nets with Learnable Up- and Downsampling* [C. Etmann, R. Ke (Presentation 17.06.)]

Fully-Learned Inversion:

- Zhu et al., "Image reconstruction by domain-transform manifold learning"
- He et al., "Radon Inversion via Deep Learning"

Unsupervised learning without data

Given measured noisy data

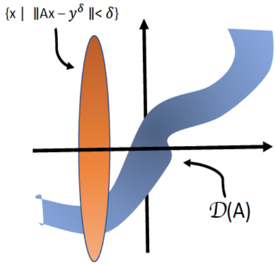
$$y^\delta = Ax^\dagger + \tau, \qquad (1)$$

Train neural network $\varphi_W(z)$ with parameters W by minimizing

$$\min_{\mathcal{W}} \|A\varphi_{\mathcal{W}}(z) - y^\delta\|^2 \qquad (2)$$

Fixed input z, single data y^δ

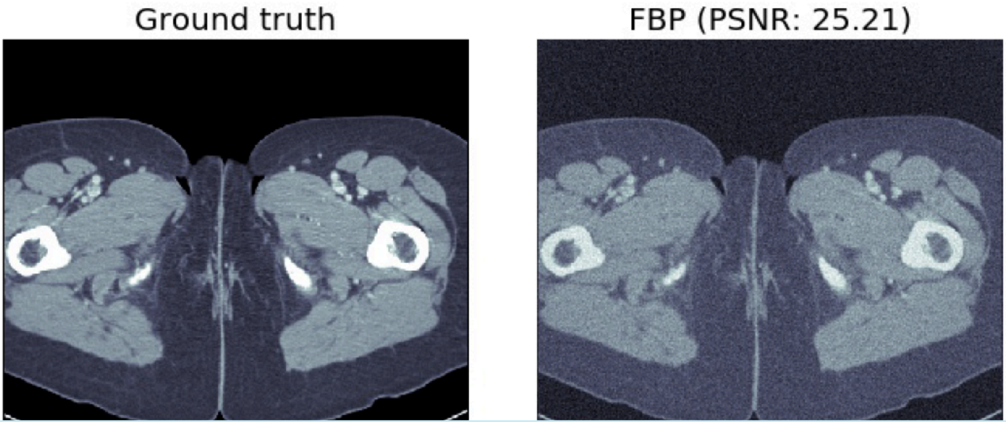
Output $\hat{x} = \varphi_{\mathcal{W}}(z)$



Dmitry Ulyanov, Andrea Vedaldi, Victor S. Lempitsky, *Deep Image Prior*, 2017, https://dblp.org/rec/bib/journals/corr/abs-1711-10925

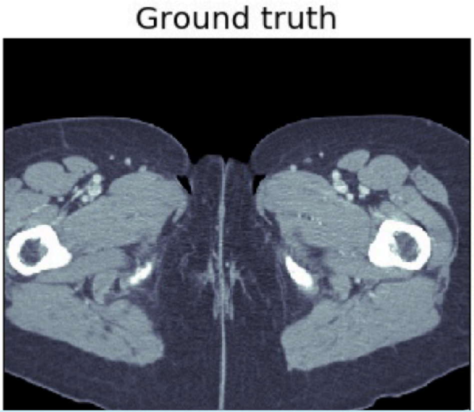
Example: Mayo data (D. Otero)

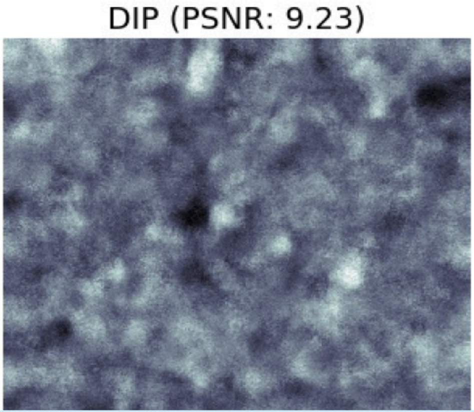
Number of angles reduced by factor 4



Example: Mayo data (D. Otero)

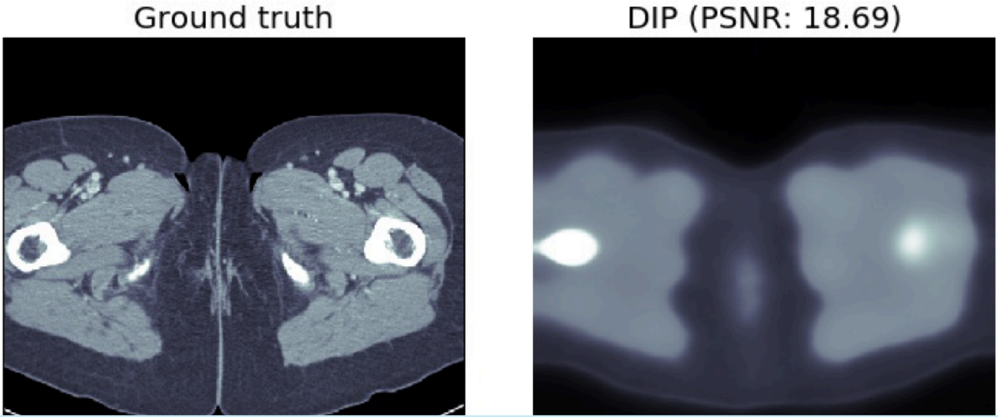
Number of angles reduced by factor 4, random initialization





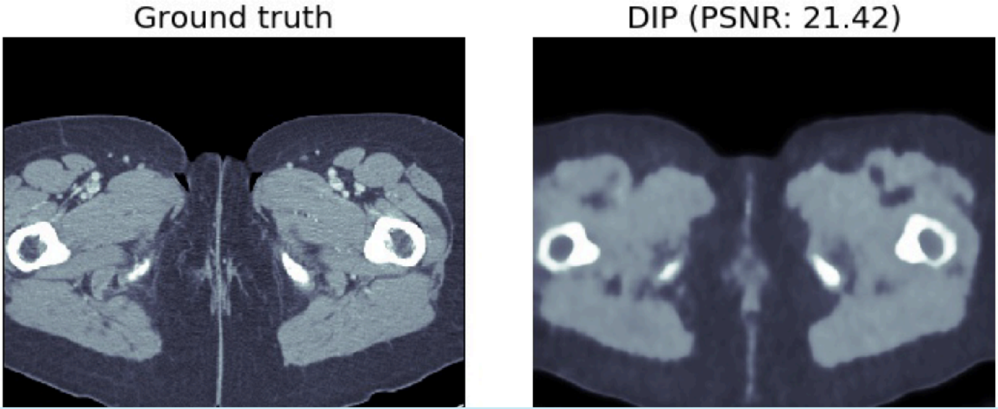
Example: Mayo data (D. Otero)

Number of angles reduced by factor 4, iteration 100



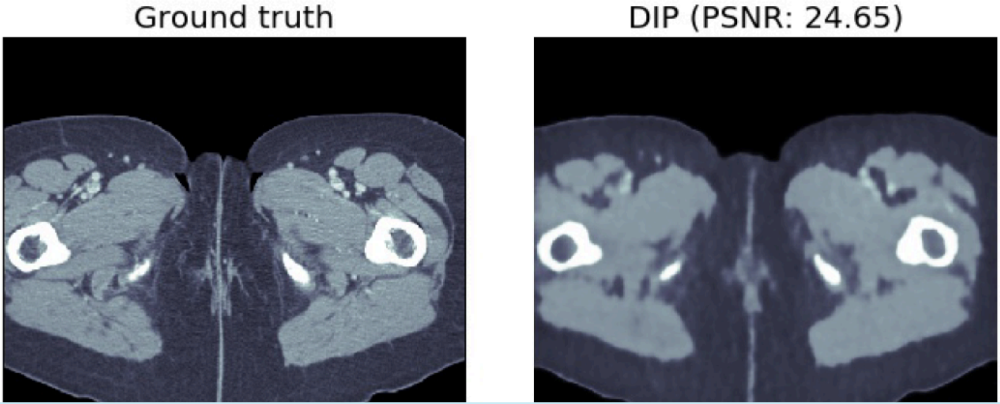
Example: Mayo data (D. Otero)

Number of angles reduced by factor 4, iteration 500



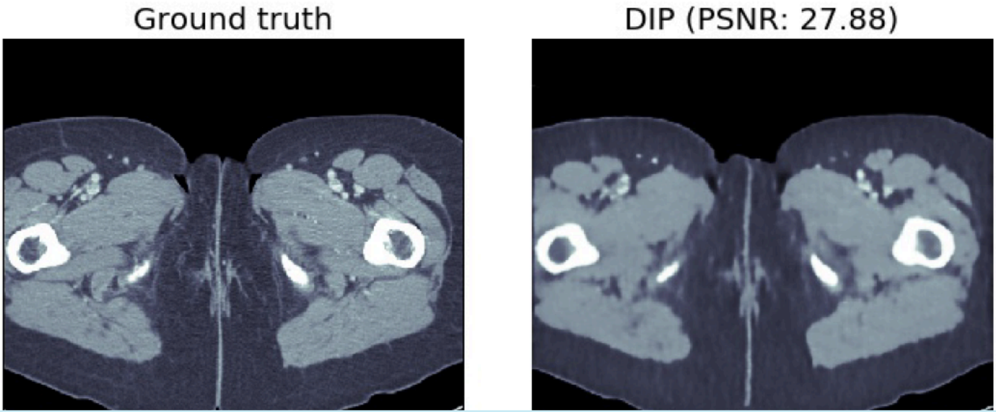
Example: Mayo data (D. Otero)

Number of angles reduced by factor 4, iteration 1000



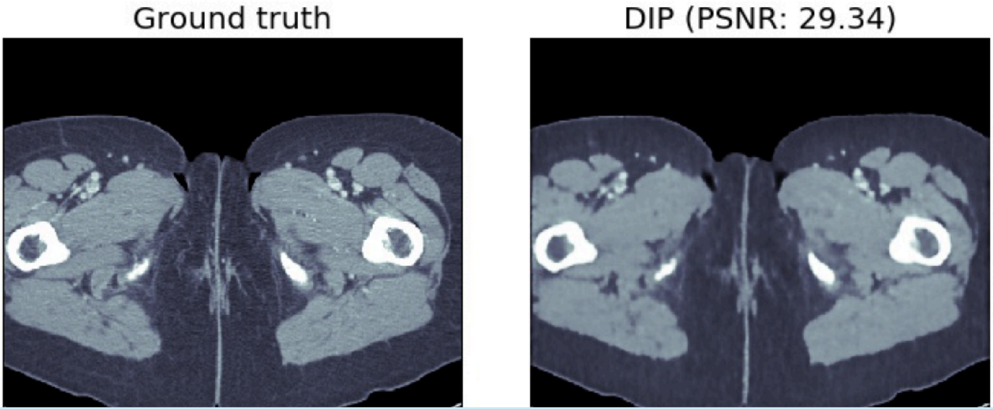
Example: Mayo data (D. Otero)

Number of angles reduced by factor 4, iteration 2000



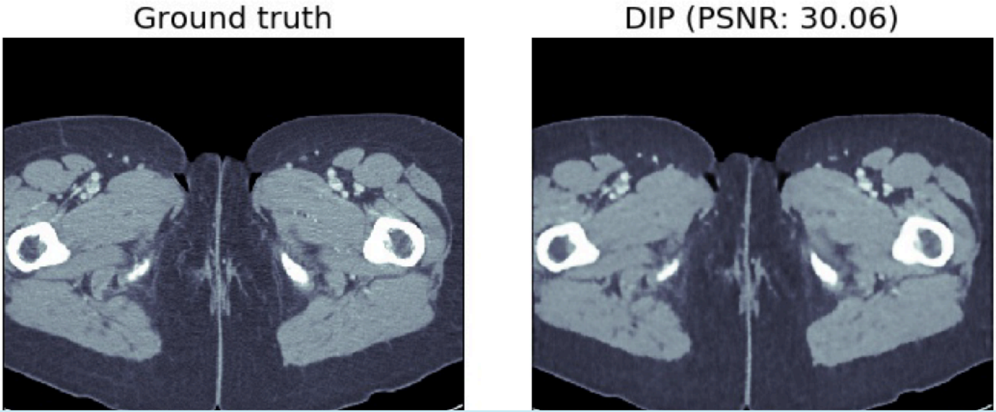
Example: Mayo data (D. Otero)

Number of angles reduced by factor 4, iteration 3000



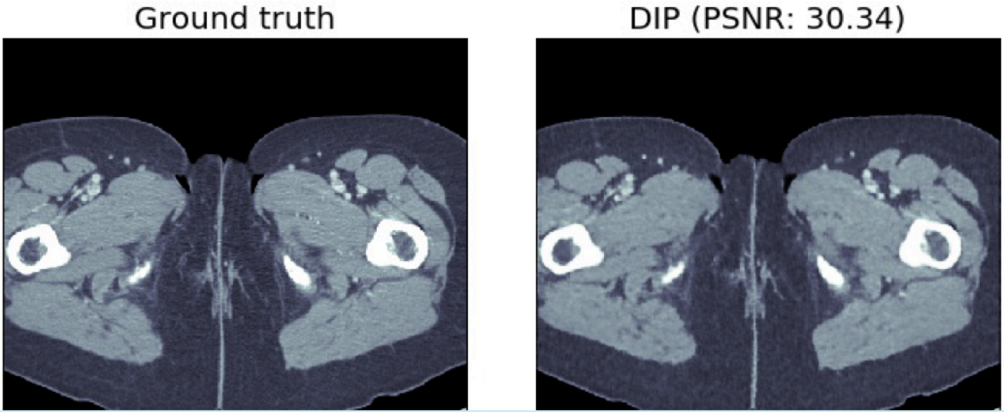
Example: Mayo data (D. Otero)

Number of angles reduced by factor 4, iteration 4000



Example: Mayo data (D. Otero)

Number of angles reduced by factor 4, iteration 5000



Analytic Deep Prior

$\varphi_{\mathcal{W}}(z)$ is identical to bi-level optimization ($W = I - \lambda B^* B$)

$$\min_{\mathcal{W}} \|A\varphi_{\mathcal{W}}(z) - y^\delta\|^2$$

$$\varphi_{\mathcal{W}}(z) \sim argmin J_B(x) = \frac{1}{2}\|Bx - y^\delta\|^2 + \alpha\mathcal{R}(x),$$

Updating W, i.e. B, changes the discrepancy term in the Tikhonov functional.

Definition

Analytic deep prior W is trained from a single data point y^δ by gradient descent for

$$\min_{W} \|A\varphi_{\mathcal{W}}(z) - y^\delta\|^2. \tag{3}$$

Lemma

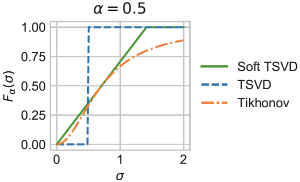
Linear inverse problems: Analytic deep prior with LISTA type network is an optimal regularization method with filter function

$$F_\alpha(\tau) = \begin{cases} 1 & \tau \leq 2\sqrt{\alpha} \\ \tau/2\sqrt{\alpha} & \tau < 2\sqrt{\alpha} \end{cases}$$

Proof: Consider

$$\psi(x, B) = prox_{\lambda\alpha R}\left(x - \lambda B^*(Bx - y^\delta)\right) - x$$

and apply implicit function theorem.



Benchmarking on Low-Dose CT

- Radiation during the scan is potentially harmful
- Reduce radiation by using lower intensity or fewer angles
  $\rightarrow$ Substantial performance decrease for classical reconstruction methods
- Learned methods show convincing results for **reconstruction** and classification on low-dose CT data
- Goal: fair comparison on real and synthetic data, easily accessible via python library DIV$\alpha\ell$

```
pip install dival
```

LoDoPaB-CT Dataset

- Based on helical thoracic CT scans from the LIDC/IDRI Database

  S.G. Armato et al., Lung Image Database Consortium (LIDC) Image Database Resource Initiative, 2011

- Around 40 000 scan slices from 800 patients
- Simulation setting:
  - Poisson noise matching 4096 photons per detector pixel before attenuation
  - Parallel beam geometry
  - 1000 projection angles
  - 513 projection beams
- Preprint available on arXiv:1910.01113

  J. Leuschner et al., The LoDoPaB-CT Dataset: A Benchmark Dataset for Low-Dose CT, 2019

Image Quality:

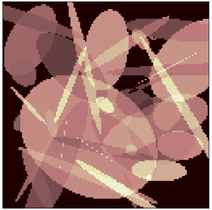
- Peak signal-to-noise ratio PSNR: pixel-wise comparison
- Structural similarity SSIM (Wang et al.): compares overall image structure

Application-related:

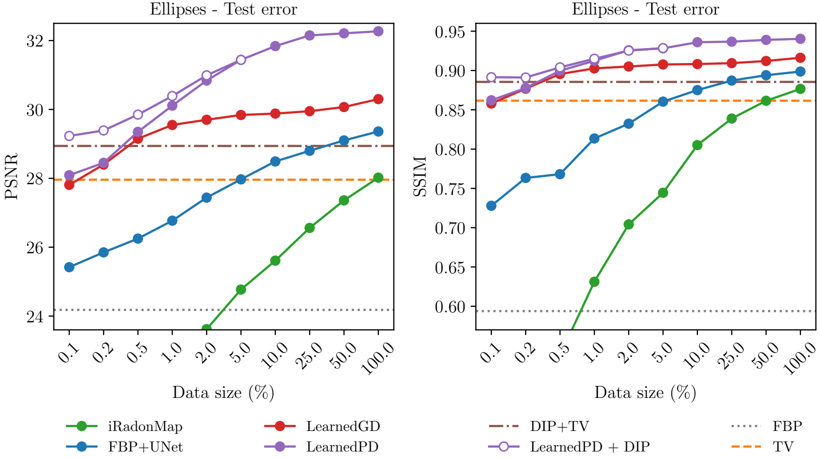
- Number of training samples
- Runtime and memory efficiency
- Number of measurement angles & dose
- Applicability for different noise levels and CT scanners
- Influence on subsequent tasks, e.g. segmentation
- Mathematical guarantees
- ...

Comparison of Reconstruction Methods (Baguer et al.)

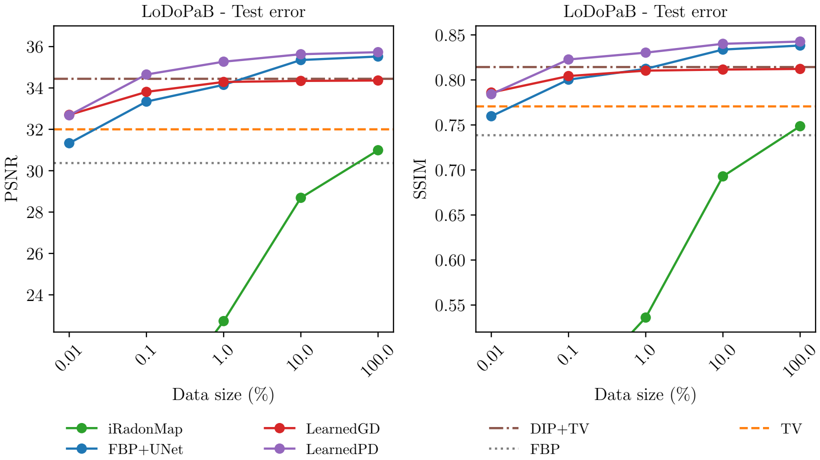
- Classical and DL methods
- PSNR, SSIM and number of training samples
- Synthetic Ellipses Dataset:
  - Random ellipse phantoms
  - Around 40 000 images in total
  - Sparse angle: Undersampled & Gaussian noise
- LoDoPaB-CT (Leuschner et al.):
  - Thoracic CT scans from the LIDC/IDRI database (Armato III et al.)
  - Over 800 patients and 40 000 scan slices
  - Simulated low photon count: Poisson noise



Synthetic Ellipses



LoDoPaB-CT


LoDoPaB - Test error
PSNR
36
34
32
30
28
26
24
0.01
0.1
1.0
10.0
100.0
Data size (%)
LoDoPaB - Test error
SSIM
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.01
0.1
1.0
10.0
100.0
Data size (%)
iRadonMap
FBP+UNet
LearnedGD
LearnedPD
DIP+TV
FBP
TV

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